

# Study & Analysis of CFO Estimation in OFDM with MIMO

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**Abstract:** Synchronization is an essential task for any digital communication system. Without a proper accurate synchronization method, it is not possible to reliably receive the transmitted data. Synchronization is the first and most important task that must be performed at the receiver. So, whole receiver architecture depends on the synchronization method that is used. Orthogonal frequency division multiplexing (OFDM) is one of the most favourable technologies deployed in fourth generation systems. It is a multicarrier transmission technique, which essentially transforms the frequency selective fading channel into flat fading channel by divides the single wideband channel into number of parallel narrowband channels. Thus, simplifies the equalizer complexity. As we know every system has its own advantages and disadvantages, OFDM also suffers from its disadvantages. One of the main disadvantages of OFDM is its sensitivity to Carrier Frequency Offset (CFO). So, accurate estimation and compensation of CFO is the major task to be performed for proper reception of data. Furthermore, when OFDM technique is used along with MIMO systems, the task of CFO estimation becomes much more difficult due to the presence of multiple antenna interference (MAI). This paper takes an overall look at this issue. In this paper, we will discuss about Chu sequences and training sequence based CFO estimation for MIMO OFDM systems which use Chu sequence based training sequences for estimation purpose.

**Keywords:** MIMO, OFDM, Carrier Frequency Offset.

## I. INTRODUCTION

As we know that OFDM is a popular method for high data rate wireless transmission. OFDM may be combined with antenna arrays at the transmitter and receiver to increase the diversity gain and/or to enhance the system capacity on time variant and frequency selective channels, resulting in a multiple-input multiple-output configuration. This combination MIMO OFDM is very beneficial as OFDM will transform each frequency selective MIMO channel into a set of parallel frequency flat MIMO channels and therefore decreases equalization complexity drastically in MIMO systems [3].

Synchronization is an essential and most important task for any digital communication system. Without a proper accurate synchronization method, it is not possible to reliably receive the transmitted data. It the same case for MIMO OFDM systems too. Failing to synchronize the carrier frequency will destroy the orthogonality between subcarriers and cause ICI. So, accurate estimation and compensation of CFO is very important. But, while considering MIMO OFDM systems there exists multi-antenna interference (MAI) between the received signals from different transmit antennas. The MAI makes CFO estimation more difficult, and a careful training sequence design is required for training-based CFO estimation in case of MIMO systems.

In this paper, we will have discussion on Chu sequences. Then after we will have a discussion on CFO estimators

for MIMO OFDM systems, which use sequences constructed from Chu sequences as their training sequence. And finally simulation results of the estimators discussed will be produced and compared.

## II. OFDM

An Orthogonal frequency division multiplexing (OFDM) is one of the most favourable technologies deployed in 4G systems as it essentially transforms the frequency selective fading channel into a flat fading channel. Even though this technology has been discovered for more than forty years, its wide spread success in commercial applications started in 1990's with the introduction of Digital Subscriber Line (DSL), which brought affordable broadband internet access to home users. OFDM was also adopted in the wireless local area network (WLAN) standards, such as IEEE 802.11a, IEEE802.11g and the upcoming IEEE802.11n. OFDM is also a potential candidate for the fourth generation mobile communication systems as it forms its physical layer standard [4].

OFDM has many advantages that satisfy our increasing demand in high bandwidth wireless communications.

- It is highly spectral efficient as it allows overlapping of subcarriers spectrum and as it divides the single frequency selective channel into number of parallel narrowband flat fading sub-channels it is highly immune to fading in wireless environment.

- It also eliminates inter-symbol interference (ISI) through the use of cyclic prefix technique.
- OFDM is computationally efficient by using FFT techniques to implement the modulation and demodulation functions. Meanwhile, as the cost of digital signal processor is dropping, OFDM systems become more affordable and easier to be implemented. OFDM is also a very flexible technology.
- It is not limited to simple single-input single-output (SISO) systems; it can be extended to multiple-input multiple-output (MIMO) systems for spatial diversity, or it can assign subsets of subcarriers to individual users for multiple access. Orthogonal Frequency Division Multiple Access (OFDMA) is one of the typical multiple access schemes derived from OFDM.

OFDM is a multicarrier transmission technique, which divides the single wideband channel into number of parallel narrowband channels called sub-channels; each subcarrier in each sub-channel is being modulated by a low rate data stream. We know that in frequency division multiplexing (FDM) technique, lot of spectrum is wasted in the form of guard bands between the adjacent channels for channel isolation and filtering purposes. In a typical system, up to 50% of the total spectrum is wasted in this manner. This problem becomes worse as the sub-channel bandwidth becomes narrower and total frequency band increases.

But, as mentioned earlier OFDM technique over comes this problem by splitting the available bandwidth into number of parallel narrowband sub-channels. The subcarriers of each sub-channel are made orthogonal to one another, allowing them to be spaced very close together with no overhead (like guard bands), as in FDM. This basic concept of OFDM saving the spectrum is illustrated in Fig.2.1.

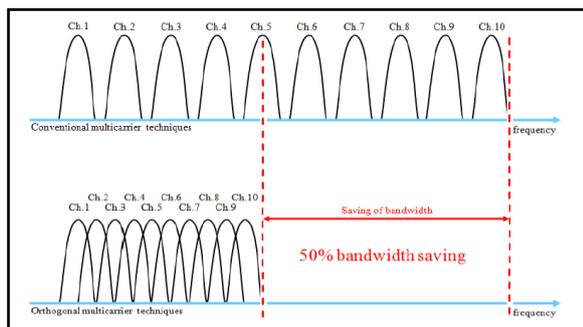


Fig.2.1 Basic concept of OFDM system

The basic steps performed in the OFDM system are shown in Fig.2.2. To generate OFDM signal, the high data rate input stream is converted into number of low data rate stream using serial to parallel converter. Each subcarrier is modulated with one of this low data rate streams in IDFT block and finally this signal is transmitted serially after adding cyclic prefix. At the receiver, exactly opposite steps are carried out as shown in Fig.2.2. Since each

parallel sub-channel is essentially low data rate channel and since it is narrowband, it experiences flat fading. This is another advantage of OFDM technique, which will reduce the complexity of equalizer at the receiver end.

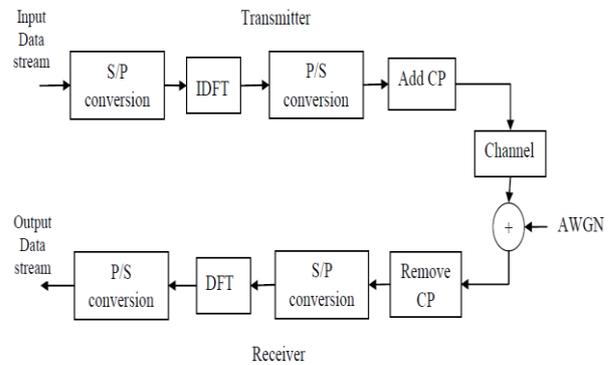


Fig.2.2 Basic steps in OFDM technique

### III.MULTIPLE INPUT MULTIPLE OUTPUT SYSTEM

Perhaps another one of the most interesting trends in wireless communication is the proposed use of multiple input multiple output systems. A MIMO system uses multiple transmitter antennas and multiple receiver antennas to break a multipath channel into several individual spatial channels. Now MIMO systems represent a huge change in how wireless communication systems are designed. This change reflects how we view multipath in a wireless system.

#### The Prospects of MIMO

From an information theoretic perspective, increasing the number of antennas essentially allows to achieve higher spectral efficiency compared to single-input single-output systems. Actual transmission schemes exploit this higher capacity by leveraging three types of partially contradictory gains:

- Array gain refers to picking up a larger share of the transmitted power at the receiver which mainly allows to extend the range of a communication system and to suppress interference.
- Diversity gain counters the effects of variations in the channel, known as fading, which increases link-reliability and QoS.[9]
- Multiplexing gain allows for a linear increase in spectral efficiency and peak data rates by transmitting multiple data streams concurrently in the same frequency band. The number of parallel streams is thereby limited by the number of transmit or receive antennas, whichever is smaller.

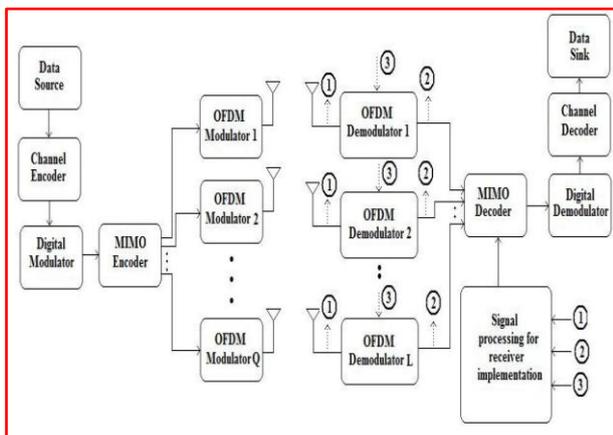
**Old Perspective:** The ultimate goal of wireless communications is to combat the distortion caused by multipath in order to approach the theoretical limit of capacity for a band-limited channel.

**New Perspective:** Since multipath propagation actually represents multipath channels between a transmitter and receiver, the ultimate goal of wireless communications is to use multipath to provide higher total capacity than the theoretical limit for a conventional band limited channel. The basic idea is to usefully exploit the multipath rather than mitigate it, considering the multipath itself as a source of diversity that allows the parallel transmission of independent data sub-streams from the same user. The exploitation of diversity and parallel transmission of several data streams on different propagation paths at the same time and frequency allows for extremely large capacities compared to conventional wireless systems. The prospect of many orders of magnitude improvement in wireless communication performance at no cost of extra spectrum (only hardware and complexity are added) is largely responsible for the success of MIMO.

**IV. MIMO-OFDM SYSTEMS**

For high data rate transmission, the multi-path characteristic of the environment causes the MIMO channel to be frequency-selective. OFDM can transform such a frequency-selective MIMO channel into a set of parallel frequency-flat MIMO channels, and therefore decrease receiver complexity. The combination of the two powerful techniques, MIMO and OFDM, is very attractive, and has become a most promising broadband wireless access scheme.

Fig.4.1 shows the simplified block diagram of  $Q \times L$  MIMO OFDM system. The source bit stream from data source is encoded by a forward error correction (FEC) encoder/ channel encoder. After that, the coded bit stream is mapped to a constellation by the digital modulator, and encoded by a MIMO encoder. Then each of the parallel output symbol streams corresponding to a certain transmit antenna follows the same transmission process. Firstly, these symbols are arranged in blocks and modulated by Inverse Discrete Fourier Transform (IDFT) or preferably Inverse Fast Fourier Transform (IFFT) to an OFDM symbol sequence.



**Fig.4.1 The simplified block diagram of (Q X L) MIMO OFDM system**

A cyclic prefix is attached to every OFDM symbol to mitigate the effect of channel delay spread, and a preamble is inserted in every slot for timing. Finally, the constructed data frame is transferred to IF/RF components for transmission.

At the receiver end, received symbol stream from IF/RF components over the receive antennas are sent to OFDM demodulator, in which cyclic prefix is removed from the received symbol stream.[8] Remaining OFDM symbol is demodulated by Discrete Fourier Transform (DFT) or Fast Fourier Transform (FFT). The output symbols of each OFDM demodulator are given to MIMO decoder to combine it into single high data rate stream. These symbols are de-mapped using digital demodulator and given to sink by performing channel decoding. There is a signal processing block for receiver implementation; this block forms a main part of the receiver. Frequency and timing synchronization and channel estimation parts are carried on received symbols and the calculated parameters are used to in estimating the received symbols correctly.

As every system has its pros and cons, OFDM also suffers from its disadvantages. One of the major disadvantages is its extreme sensitivity to carrier frequency offset (CFO). Failing to synchronize the carrier frequency will destroy the orthogonality between subcarriers [9] and cause inter-carrier interference (ICI). So, accurate estimation and compensation of CFO is very important. Therefore, many synchronization schemes have been proposed for SISO OFDM systems. Some schemes are pilot based and some are based on blind estimation. Similar to SISO OFDM systems, MIMO OFDM systems are also sensitive to CFO, moreover for MIMO-OFDM, there exists multi-antenna interference (MAI) between the received signals from different transmit antennas. The MAI makes CFO estimation more difficult, and a careful training sequence design is required for training-based CFO estimation in case of MIMO systems.

**4.1 System Model**

Let us consider the MIMO OFDM system with  $N_T$  transmit antennas,  $N_R$  receive antennas and  $N$  subcarriers. Fig.4.2 shows the detailed block diagram of  $N_T \times N_R$  MIMO OFDM system. Suppose the training from sequence transmitted the  $\square$ th  $\square$  transmit antenna be denoted by  $N \times 1$  vector  $t_{\square}$ . Before transmission, this vector is processed by an IDFT and a cyclic prefix of length  $N_g$  is inserted. Length of  $N_g$  is selected such that it is greater than length of the channel  $L$ . i.e.  $N_g \geq L \geq 1$ . If we assume that all transmit-receive antenna pairs are affected by the same relative CFO ' $\epsilon$ '. [5] Then, the received vector  $y_{\square}$  at the  $\square$ th  $\square$  receive antenna after removing cyclic prefix of length  $N \square 1$  is given by

$$\bar{y}_v = \sqrt{N} e^{j2\pi\epsilon N_g / N} \mathbf{D}_N(\epsilon) \sum_{\mu=0}^{N_T-1} \left\{ \mathbf{F}_N^H \text{diag} \left\{ \bar{h}^{(v,\mu)} \right\} \bar{t}_{\mu} \right\} + \bar{w}_v \tag{4.1}$$

where,

$$D_N(\varepsilon) = \text{diag} \left\{ \left[ 1, e^{j2\pi\varepsilon/N}, \dots, e^{j2\pi\varepsilon(N-1)/N} \right]^T \right\}$$

$F_N^H$  is Hermitian of Fast Fourier Transform matrix.

$[F_N]_{M,N} = \sqrt{N} \exp(-j2\pi mn/N)$  is the (m,n)th entry of matrix  $F_N$ .

$$\bar{h}^{(v,\mu)} = F_N \left[ \bar{e}_N^0, \bar{e}_N^1, \dots, \bar{e}_N^{L-1} \right] \bar{h}^{(v,\mu)}$$

$\bar{h}^{(v,\mu)}$  is  $L \times 1$  vector denoting length- $L$  channel impulse response from the  $\mu$ th transmit antenna to the  $v$ th receive antenna.  $\bar{t}$  denotes training sequence vector from the  $\mu$ th antenna of length  $N \times 1$ .  $w$  denotes  $N \times 1$  vector of additive white complex Gaussian noise with zero mean and  $\sigma^2$  variance. In the subsequent sections we will use this system model equation (i.e. Eq.(4.1)) for the analysis of the CFO estimators of MIMO OFDM system which use the training sequences that are constructed using Chu sequence.

estimator and uses the two types of suboptimal training sequences that are constructed using Chu sequence. The constant time domain magnitude of Chu sequence precludes peak-to-average power ratio problems that plague many CP based systems. In addition, certain linear phase shifts when introduced into Chu sequence also will not affect the zero autocorrelation property.

Let denote the Chu sequence of length ‘P’ and each element of sequence be denoted by

$$[\bar{s}]_p = e^{j\pi L p^2 / P} \quad 0 \leq p \leq P-1$$

where  $L$  is co-prime to  $P$

$$\bar{s}_\mu = \sqrt{Q/N_T} F_P \bar{s}^{(\mu M)} \quad (4.3)$$

where,  $Q = N/P$ ,  $M = P/N_1$  and  $N_1 > N_T$  are design parameters.  $F_P$  is FFT matrix of size- $P$ . Let us refer to the training sequence constructed in Eq.(4.3) as Chu sequence based Training sequence 0 (CBTS0) and the training sequence constructed using Eq.(4.4) as Chu sequence based Training sequence 1 (CBTS1).

$$\bar{s}_\mu = \sqrt{Q/N_T} F_P \bar{s} \quad (4.4)$$

Then, the training sequence vector at the  $\mu$ th transmit antenna is constructed from Eq.(4.2) as follows:

$$\bar{t}_\mu = \Theta_{i_\mu} \bar{s}_{i_\mu} \quad (4.5)$$

$0 \leq i_\mu \leq Q - 1$  and  $i_\mu = i_\mu$ , if  $\mu = \mu$

$$\Theta_q = \left[ \bar{e}_N^q, \bar{e}_N^{q+Q}, \dots, \bar{e}_N^{q+(P-1)Q} \right]$$

And  $e_{-N}^k$  denotes the  $k$ th column vector of  $I_N$ .

According to the construction of the training sequences and exploiting the properties of Chu sequence, we note that the proposed training sequences have the following properties:

1. The training sequences are orthogonal to each other
2. All the ‘P’ pilots relevant to each training sequence are uniformly spaced.
3. The time domain sequences corresponding to the proposed frequency domain training sequences have constant amplitude and zero auto-correlation.

### 4.3 Simplified CFO Estimation using Chu sequence based Training sequence (CBTS)

We know that the maximum Likelihood CFO estimation method is computationally complicated since it requires a large point Discrete Fourier transform operation and time consuming line search. So many reduced complexity

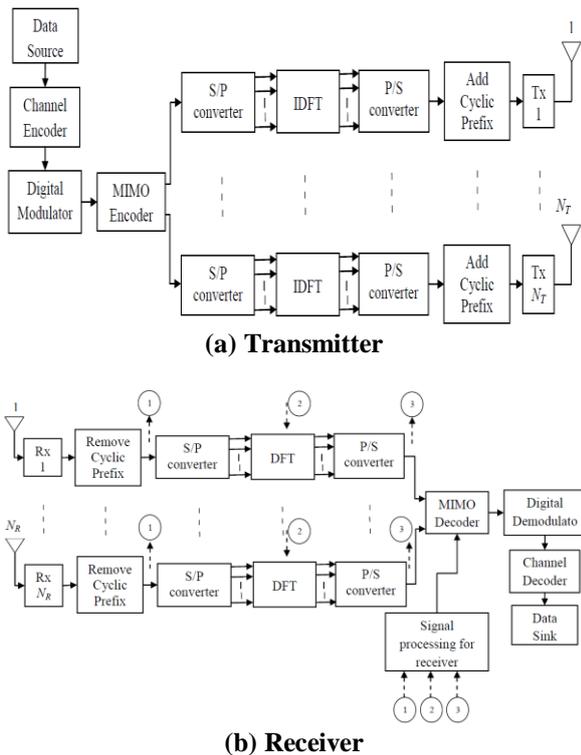


Fig.4.2 Block diagram of  $N_T \times N_R$  MIMO OFDM System

### 4.2 CFO Estimation using Chu sequence based Training sequences (CBTS)

In this section we will discuss one of the effective CFO estimation methods for MIMO OFDM system. This method will estimate the CFO in two parts; that is integer part known as ICFO and fractional part known as FCFO [11]. This estimation method is a training sequence based

algorithms have been proposed in the literature. The one which we have discussed in the section 4.4 is based on the roots directly from the cost function and need the complicated polynomial rooting operation, which is hard to implement in practical OFDM systems [6].

In this section we will discuss the CFO estimation method which uses a simple polynomial factor method instead of complicated polynomial rooting operation. This method can be implemented via a simple additions and multiplications in practical OFDM system. If we look at the received signal of the MIMO OFDM system Eq.(4.3) and by exploiting the properties of Chu sequences we can write the received vector  $y$  into the  $Q \times N_R P$  matrix  $Y$ .

$$Y = [ Y_0, Y_1, \dots, Y_v, \dots, Y_{N_r-1} ]$$

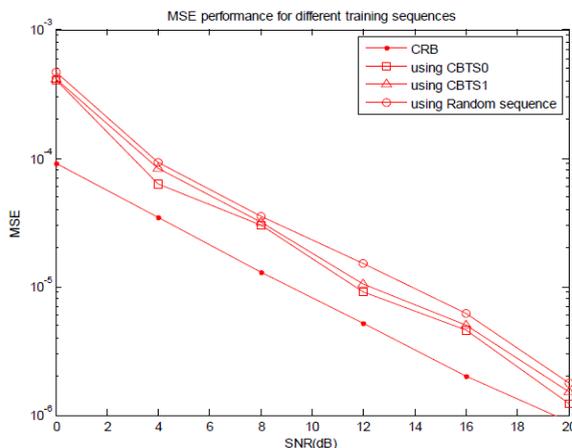
where its elements are given by

$$[Y_v]_{q,p} = \left[ \left( \bar{e}_{N_r}^T \otimes I_N \right) \bar{y} \right]_{qP+p}$$

### V. SIMULATION & RESULTS

**A.** To perform CFO estimation using Chu sequence based training sequences for MIMO OFDM system as discussed in section 4.2, we use the following parameters;

- Assumption: All transmit/receive antenna pairs are affected by same CFO
- No. of Transmit antennas :  $N_T = 3$
- No. of Receive antennas :  $N_R = 2$
- No. of subcarriers :  $N = 1024$
- Guard interval :  $N_g = 64$
- Training sequence used : Random sequence and Chu sequence based training sequences (CBTS0 & CBTS1)
- Training sequence parameters :  $P = 64$  and  $Q = 16$
- Each channel has: 4 independent Rayleigh fading taps whose average powers and propagation delays are  $\{0, -9.7, -19.2, -22.8\}$  dB and  $\{0, 0.1, 0.2, 0.4\}$  us respectively.
- Modulation scheme used : QAM modulation



**Fig.5.1** MSE performance of 3 X 2 MIMO OFDM system using different training sequences for CFO estimation

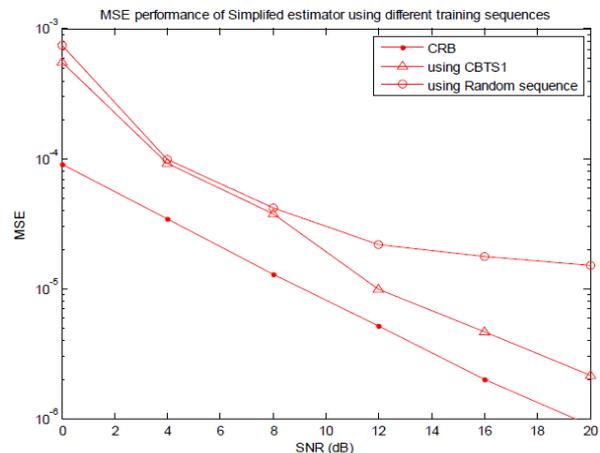
- SNR values taken:  $\{0, 5, 10, 15, 20\}$  dB
- No. of Monte Carlo simulation trials:  $10^6$ .

Fig.5.1 shows the Mean Square Error (MSE) results of the estimation method discussed in section 4.2 using different types of training sequences. We can note from the plot that the performance of estimator using Chu sequence based training sequence is better than random sequence. If we compare CBTS0 and CBTS1 performance, CBTS0 performance is little better than CBTS1 and approaches CRLB.

**B.** To perform CFO estimation using Chu sequence based training sequences for MIMO OFDM system using simplified estimator as discussed in section 4.3, we use the following parameters;

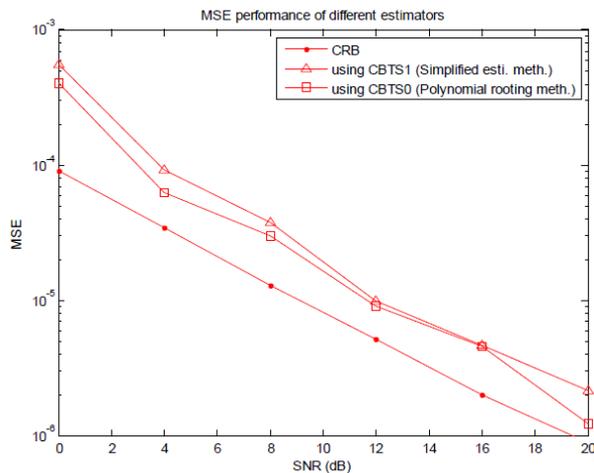
- Assumption : All transmit/receive antenna pairs are affected by same CFO
- No. of Transmit antennas :  $N_T = 3$
- No. of Receive antennas :  $N_R = 2$
- No. of subcarriers :  $N = 1024$
- Guard interval :  $N_g = 64$
- Training sequence used: Random sequence and Chu sequence based training Sequence (CBTS1)
- Training sequence parameters :  $P = 64$  and  $Q = 16$
- Each channel has: 4 independent Rayleigh fading taps whose average powers and propagation delays are  $\{0, -9.7, -19.2, -22.8\}$  dB and  $\{0, 0.1, 0.2, 0.4\}$  us respectively.
- Modulation scheme used : QAM modulation
- SNR values taken :  $\{0, 5, 10, 15, 20\}$  dB
- No. of Monte Carlo simulation trials : 106

Fig.5.2 shows the Mean Square Error (MSE) results of the simplified estimation method used for MIMO OFDM system that is discussed in section 4.3 using different types of training sequences. We can note from the plot that the performance of estimator using Chu sequence based training sequence is better than random sequence.



**Fig.5.2** MSE performance of 3 X 2 MIMO OFDM system with simplified estimator using different training sequences

Fig.5.3 shows the MSE performance of the both estimation methods i.e. polynomial rooting method (discussed in section 4.2) and computationally simplified estimation method (discussed in section 4.3) using CBTS. We can see that the performance of computationally simplified estimation method is close to the one which is discussed in section 4.2.



**Fig.5.3 MSE performance of 3 X 2 MIMO OFDM system using different estimators using Chu sequence based training sequences (CBTS)**

### VI. CONCLUSION

Synchronization is an essential and most important task for any digital communication system. Without a proper accurate synchronization method, it is not possible to reliably receive the transmitted data. It the same case for MIMO OFDM systems too. Failing to synchronize the carrier frequency will destroy the orthogonality between subcarriers and cause ICI. So, accurate estimation and compensation of CFO is very important. But, while considering MIMO OFDM systems there exists multi-antenna interference (MAI) between the received signals from different transmit antennas. The MAI makes CFO estimation more difficult, and a careful training sequence design is required for training-based CFO estimation in case of MIMO systems.

In this paper, we have discuss on CFO estimation technique for MIMO OFDM systems which use Chu sequence based training sequences for the estimation purpose. In which we can see also that the performance of computationally efficient method is efficient and which avoids complex polynomial rooting operation present in the earlier mentioned Chu sequence based estimation method.

### REFERENCES

[1] A Tomasoni, G Gatti, S Bellini, M Ferrari, M Siti, "Efficient OFDM channel estimation via an information criterion". IEEE Trans. Wireless Commun. 12(3), 1352–1362 (2013).

[2] A. Stefanov and T. Duman, "Performance bounds for turbo-coded multiple antenna systems," IEEE J. Select. Areas Commun., vol. 21, pp. 374–381, Apr. 2003.

[3] B. Hassibi and B. Hochwald, "High-rate codes that are linear in space and time," IEEE Trans. Inform. Theory, vol. 48, pp. 1804–1825, July 2002.

[4] Brad W. Zarikoff, James K. Cavers "Multiple Carrier Frequency Offset and Channel State Estimation in the Fading Channel" 2008 IEEE 68th Vehicular Technology Conference.

[5] C. Yu, Y. Lee, "An OFDM Carrier Frequency Offset Estimation Scheme with Wide Fractional Offset Estimation Range" Vol. 11, April 2013

[6] DKC So, RS Cheng, "Iterative EM receiver for space-time coded systems in MIMO frequency-selective fading channels with channel gain and order estimation". IEEE Trans. Wireless Commun. 3(6), 1928–1935 (2004)

[7] Hao Zhou, Amaresh V. Malipatil and Yih-Fang Huang "Maximum-Likelihood Carrier Frequency Offset Estimation for OFDM Systems in Fading Channels" Department of Electrical Engineering University of Notre Dame.

[8] J Siew, J Coon, A Nix, M Beach, S Armour, J McGeehan, "A joint channel and channel length estimation algorithm for MIMO-OFDM" in Proc. Commun. Wireless Syst. Conf. Sept 20–22. (Mauritius, 2004)

[9] Jian Li, Guoqing Liu, and Georgios B. Giannakis, Fellow, IEEE "Carrier Frequency Offset Estimation for OFDM-Based WLANs" IEEE Signal Processing Letters, Vol. 8, No. 3, March 2001

[10] Praween Kumar Nishad "Carrier Frequency Offset Estimation in OFDM Systems" IEEE transactions on communications.

[11] Saeed Mohseni and Mohammad A. Matin "Study Of The Sensitivity Of The Ofdm Wireless Systems To The Carrier Frequency Offset (Cfo)" International Journal of Distributed and Parallel Systems (IJDPS) Vol.4, No.1, January 2013

[12] Tsui-Tsai Lin and Fuh-Hsin Hwang "On the CFO/Channel estimation technique for MIMO-OFDM systems without using a prior knowledge of channel length" Lin and Hwang EURASIP Journal on Wireless Communications and Networking (2015).

[13] W. Su, Z. Safar, M. Olfat, and R. Liu, "Obtaining full-diversity space-frequency codes from space-time codes via mapping," IEEE Trans. Signal Processing, vol. 51, pp. 2905–2916, Nov. 2003.

[14] Yinsheng LIU, Zhenhui TAN "Carrier Frequency Offset Estimation for OFDM Systems using Repetitive Patterns" Radio engineering, Vol. 21, No. 3, September 2012.

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